

Asset-Liability Management 2019

Project

Introduction

The project is designed to be solved in groups of 2-3 students. The technical work should result in a report of not more than 10 pages. The report should be in such a form that a non-mathematical person gets an idea of what you have done and why. Please place important results, tables and graphs into your report, to make it accessible to a reader who does not want to dive into programs.

You may write your report in English or Portuguese.

You may contact me with questions at the address walther.neuhaus@zabler-neuhaus.no.

The deadline for delivery of the report and underlying workings is Monday, 14 October 2019.

Have fun!

You are the actuary of a pension fund and it's January 1.

Pensions are paid on 31 December from age 66 until death. No pensions are paid after age 100.

The level of pensions is EUR 10,000 this year and indexed by 2% per annum thereafter.

At present the fund has the following living pensioners and aspirant pensioners:

Age on 1.1	Females	Males
65	13	15
66	38	24
67	32	1
68	21	24
69	18	10
70	19	46
71	9	54
72	6	7
73	14	25
74	10	19

Pensioners who are 65 now and will turn 66 before 31.12 (13 females and 15 males) will receive their first payment of EUR 10,000 at the end of the year, if they are still alive. Similarly for the other age groups.

Your task is to determine assets that finance and immunise the expected liability cash outflow.

1. Expected liability cash outflow

You are using a mortality table based on 70% of a Gompertz-Makeham formula.

- Calculate the expected number of survivors at the end of the year, and every year thereafter until the last pensioner has turned 100 years.
- Calculate the expected liability cash outflow. The first payment is at the end of this year, EUR 10,000 to every pensioner who is still alive on New Year's Eve.

2. Present value, duration and convexity

The regulator EIOPA has handed down a risk-free yield curve (RFR).

- Calculate the present value of the expected liability cash flow, using that yield curve.
- Calculate the duration and convexity of the expected liability cash flow.

3. Matching and immunisation

Being risk averse, you want to match the expected liability cash flow with government bonds. Unfortunately, the asset manager informs you, the market for government bonds is liquid only for bonds with maturities of 1-15 years. Of longer maturity bonds, the only ones available have maturities 20, 25, 30 and 35 years. Assume for the sake of simplicity that all government bonds have an annual coupon rate of 3% that is paid on 31.12 of each year.

The asset manager suggests the following strategy to you:

Why don't you match the cash flow of the first 15 years with bonds that I can buy for you, and duration-match the cash flow of the later years by some suitable long-maturity bonds? In that way, at least we're safe for now.

Please do what the asset manager has suggested:

- Determine a portfolio of bonds, the payouts of which match the expected liability cash flow of the first 15 years.
- Suggest a portfolio of long-maturity bonds that matches the present value and duration of the expected liability cash flow beyond 15 years. Remember that the asset convexity ought to be at least as large as the liability convexity.
- Make a table showing: the expected liability cash flows, split between short and long and in total; the asset cash flow that will result if you buy the bonds you have found, split between short and long and in total. For each partial and total cash flow, compute its present value, duration and convexity, as a check that your results are correct. Here is a sample table:

Time t	RFR	Price of €1	Liability cash flow			Asset cash flow		
			Short	Long	Overall	Short	Long	Overall
1	-0.333556 %	1.0033	3 968 033		3 968 033	3 968 033	326 989	4 295 022
...
35	2.092944 %	0.4807		8 524	8 524		7 154 238	7 154 238
	Total		52 958 095	18 903 963	71 862 058	52 958 095	20 565 053	73 523 149
	Present value		50 496 535	14 305 052	64 801 587	50 496 535	14 305 052	64 801 587
	Duration		7.3020	20.3374	10.1796	7.3020	20.3374	10.1796
	Convexity		70.8629	427.8752	149.6739	70.8629	531.0571	172.4515

4. Presentation

Present your results in a report, setting out what you have done and (importantly!) why.

5. What if this and what if that?

ALM is never finished with one answer. Expect your audience to ask “but what if?”

Organise your calculations in a way that makes it easy to vary the following:

- a) Mortality assumptions
- b) The number of pensioners
- c) The level and indexation rate of pensions
- d) The yield curve
- e) The coupon rates of available bonds

If you’re using Excel, only assumptions may be hard-coded. Every other cell ought to contain an auditable formula. Copy-Paste-Value to transfer results between calculations is strictly forbidden.

You may also use R. Using R does not relieve you of the obligation to produce readable output, the kind of output that we take for granted in Excel; showing amounts with a fixed number of decimals and 1000-separators in amount items is a minimum courtesy to the reader.

To demonstrate that you have made a flexible calculation machine, show the results that would occur if (simultaneously)

- The mortality assumption is changed from 70% to 60% of Gompertz-Makeham; and
- Indexation is reduced from 2% to 1%.

It’s sufficient that you present the table of question 3.c with updated values.

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